

# A scalable static analysis framework for reliable program development exploiting incrementality and modularity

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Isabel García Contreras

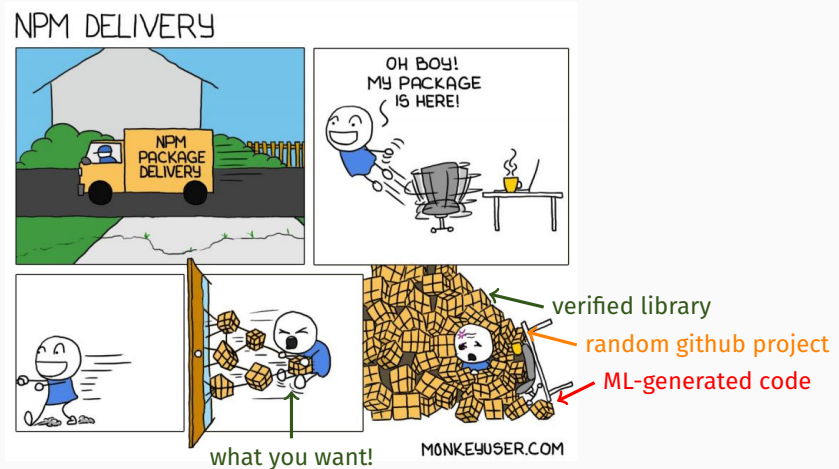
Universidad Politécnica de Madrid  
IMDEA Software Institute



PhD Thesis Defense  
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# Why analyze/verify software?

A motivation for us programmers



# Introduction

**Context:** analyzing/verifying software projects **during development** to:

- **detect and report bugs** as early as possible (e.g., on-the-fly, at commit, ...),
- **optimize** code and libraries globally for the program being developed.

**Problem:** context-sensitive analysis can be quite **precise** but also **expensive**, specially for **interactive** uses.

**Challenges** addressed in this thesis:

- **performance:** incremental and modular analysis to take advantage of **localized changes**,
  - application: *on-the-fly* assertion checking,
- **precision:** allowing the programmer to **guide** the analysis,
- **incomplete** code: manual specification + reanalysis, and
- **precision:** incomplete abstract interpretations.

# Motivation – (incremental) static *on-the-fly* verification

```
42     P = B
43     ; rewrite clause(H,B), clause(H,P), I, G, Info
44     ).
45
46 rewrite clause(H,B), clause(H,P), I, G, Info :-
47     numbertvars_2(H, 0, Lhv),
48     collect_info(B, Info, Lhv, _X, _Y),
49     add_annotations(Info, P, I, G), !.
50
51 :- pred add_annotations(Info, Phrase, Ind, Gnd)
52     : (var(Phrase), indep(Info, Phrase))
53     => (ground(Ind), ground(Gnd)).
54
55 add_annotations([], [], _).
56 add_annotations([I|Is], [P|Ps], Indep, Gnd)
57     add_annotations(I, P, Indep, Gnd),
58     add_annotations(Is, Ps, Indep, Gnd).
59
60 add_annotations(Info, Phrase, I, G) :- !,
61     para_phrase( Info, Code, Type, Vars, I, G),
62     make_CGE_phrase( Type, Code, Vars, PCode, I, G),
63     (
64         var(Code), !,
65         Phrase = PCode
66     ;
67         Phrase = Code
68     ).
69
```

Verified assertion:  
:- check calls add\_annotations(Info, Phrase, Ind, Gnd)  
: ( var(Phrase), indep(Info, Phrase) ).

Verified assertion:  
:- check success add\_annotations(Info, Phrase, Ind, Gnd)  
)  
: ( var(Phrase), indep(Info, Phrase) )

\*actual in-Emacs footage

## Constraint Logic Programs/Horn Clauses

$$\text{Head}_k \leftarrow B_{k,1}, \dots, B_{k,n_k}$$

We use **Prolog** syntax (“:-” instead of “ $\leftarrow$ ”):

```
1 list([]).           % fact
2 list([X|Xs]) :-    % rule head
3   list(Xs).        % rule body
```

## Abstract Interpretation

[Cousot & Cousot POPL'77]

Simulates the execution of the program using an **abstract domain**  $D_\alpha$ , simpler than the concrete one. Guarantees:

- analysis termination, provided that  $D_\alpha$  meets some conditions,
- results are **safe approximations** of the concrete semantics.

# Concrete semantics – AND trees

```
1 par([], P, P).  
2 par([C|Cs], P0, P) :- ←  
3   xor(C, P0, P1),  
4   par(Cs, P1, P).
```

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5 xor(0,0,0).  
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Top-down AND tree of `:- par([0, 1], 0, P).` [Bruynooghe JLP'91]

```
true  
par([0, 1], 0, P) }query  
par([C|Cs], P0, P) :- }head
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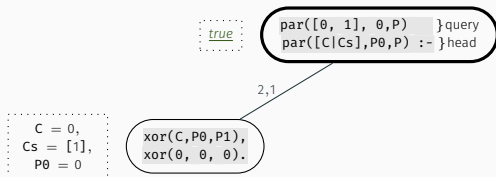
`C = 0,`  
`Cs = [1],`  
`P0 = 0`

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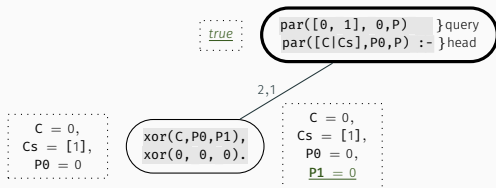


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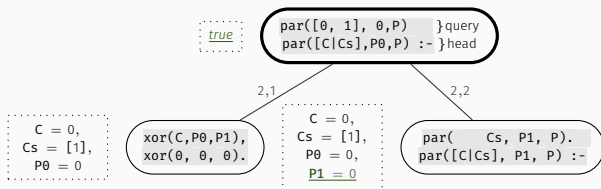
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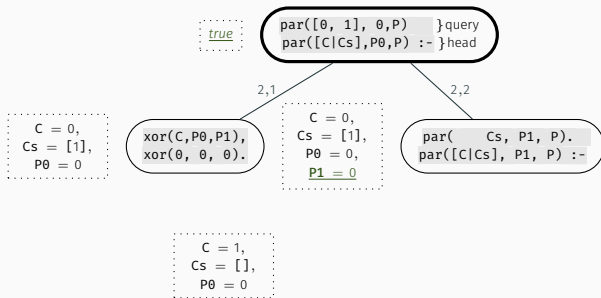
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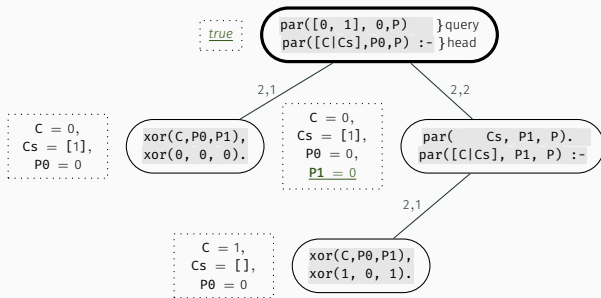
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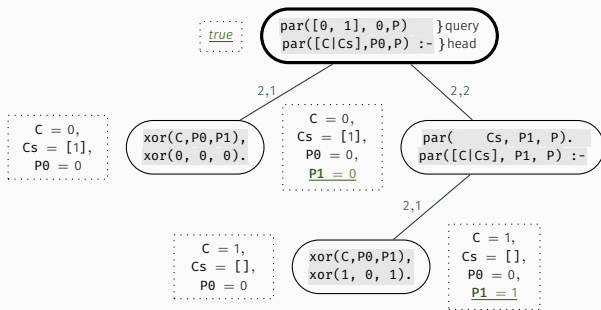


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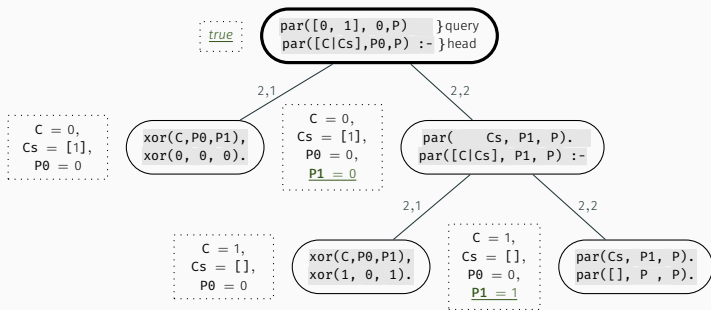


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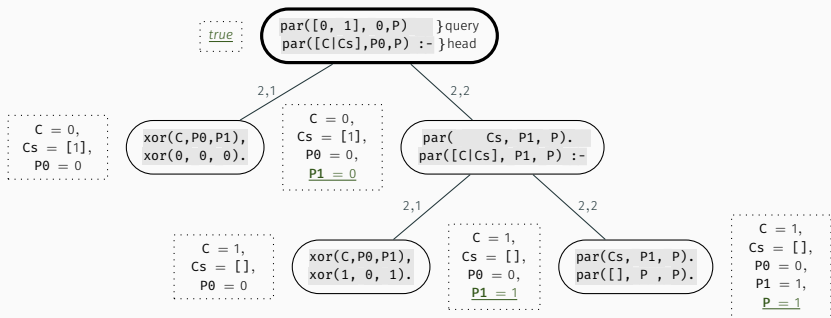


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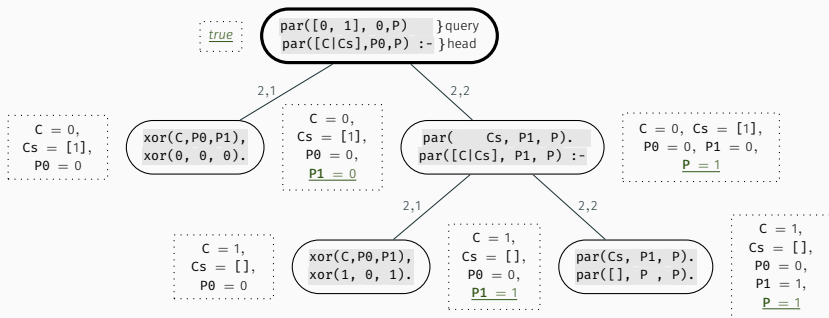


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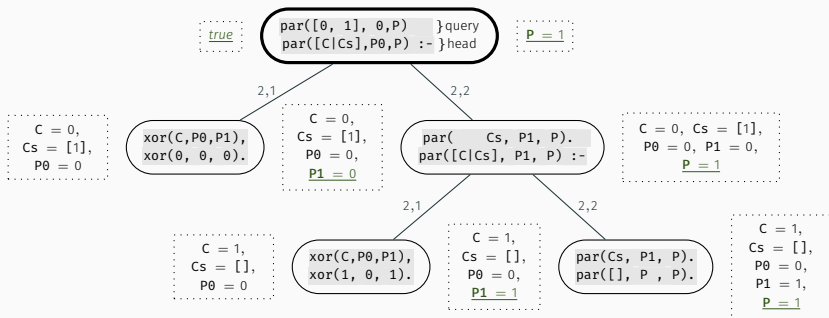


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# Abstract semantics

A PLAI **analysis graph** ( $\mathcal{A}$ ) has a set of **nodes**  $\langle A, \lambda^c \rangle \mapsto \lambda^s$  for every potentially reachable predicate, where: [NACLP'89, TOPLAS'00]

- $A$  is an atom, the predicate identifier,
- $\lambda^c$  is an abstract call to  $A$ , and
- $\lambda^s$  is the abstract answer for  $A$  and  $\lambda^c$  if any call succeeds.

$\lambda^c$  and  $\lambda^s$  are values of some abstract domain  $D_\alpha$ .

## Example

```
1 par([], P, P).
2 par([C|Cs], P0, P) :-
3     xor(C, P0, P1),
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5
6 xor(0,0,0).
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8 xor(1,0,1).
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```

Example nodes:

$\langle \text{par}(L, P0, P), \top \rangle \mapsto (P0/bit, P/bit)$

*For any call to par that succeeds, P0 and P are either 1 or 0.*

$\langle \text{par}(L, P0, P), (P0/-) \rangle \mapsto \perp$

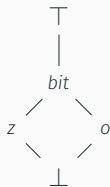
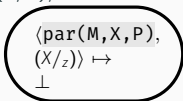
*If par is called with P0 a negative number, it always fails.*

# Building an analysis graph

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Initial query  $\langle \text{par}(M, X, P), (X/z) \rangle$



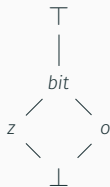
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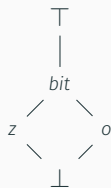
$\langle \text{par}(M, X, P), (X/z) \rangle \mapsto (X/z, P/z)$



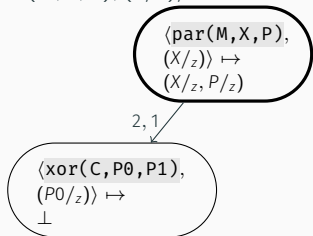
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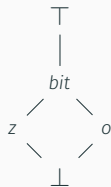
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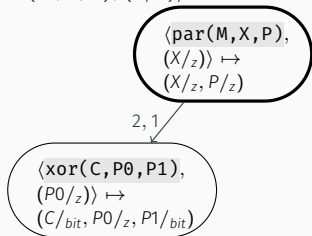
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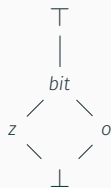
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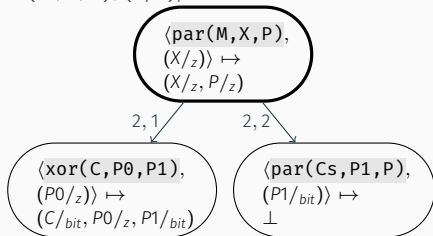
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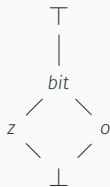
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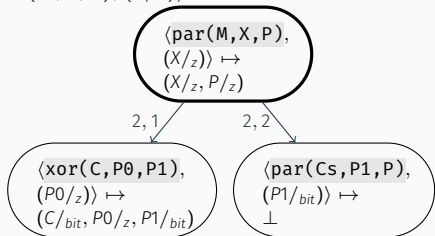
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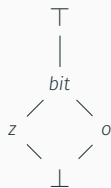




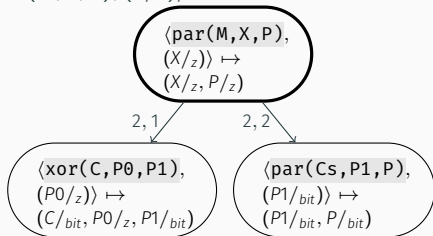
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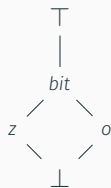
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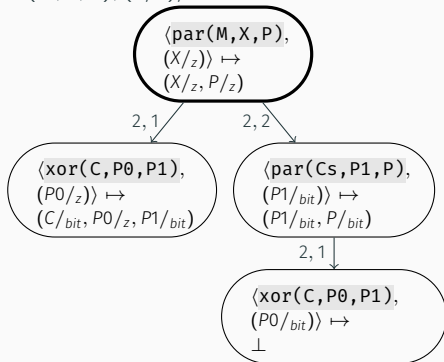
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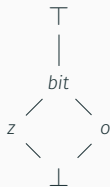
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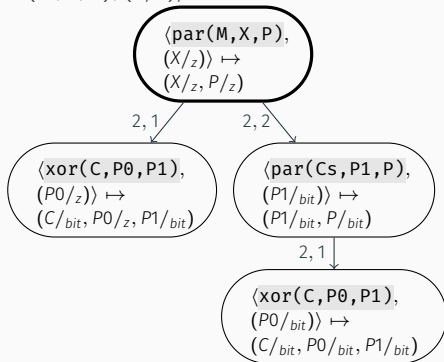
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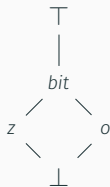
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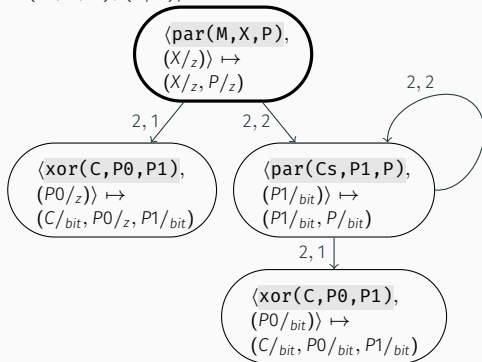
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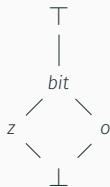
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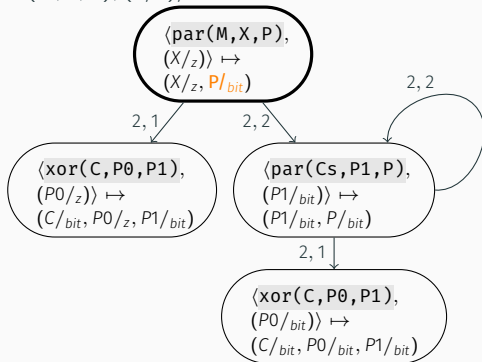
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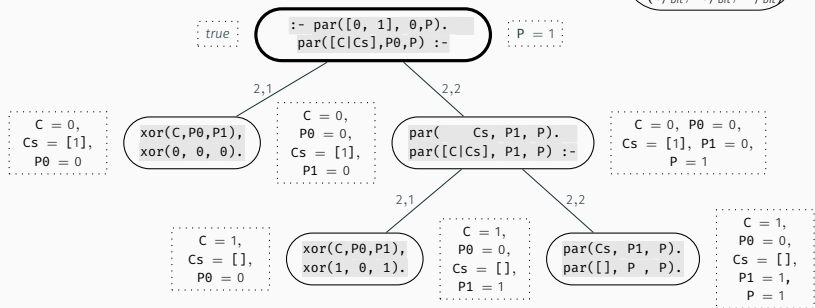
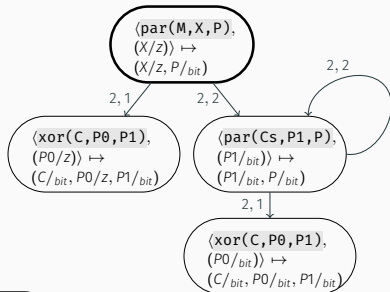


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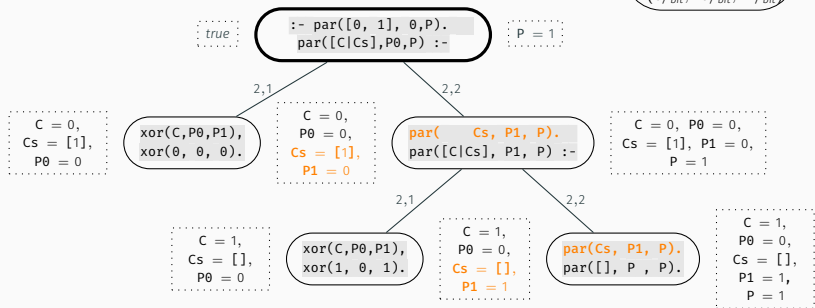
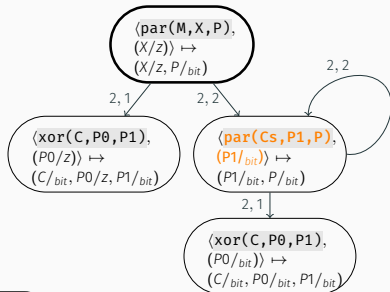
# Analysis correctness

An analysis graph  $\mathcal{A}$  is **correct** for a program  $P$  and a set of queries  $Q$  if it approximates all the calls, answers, and dependencies in the concrete semantics  $\llbracket P \rrbracket_Q$  (a set of AND trees).



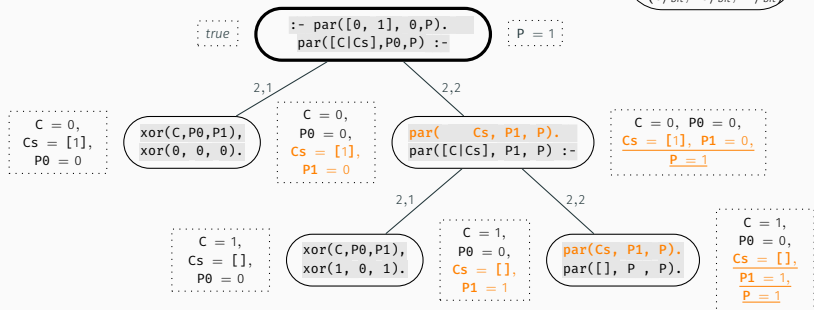
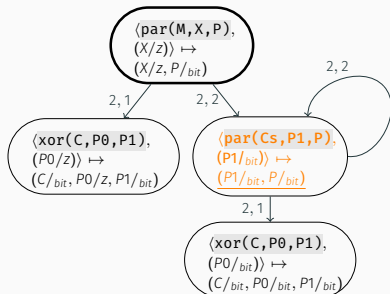
# Analysis correctness

An analysis graph  $\mathcal{A}$  is **correct** for a program  $P$  and a set of queries  $Q$  if it approximates all the **calls**, answers, and dependencies in the concrete semantics  $\llbracket P \rrbracket_Q$  (a set of AND trees).



# Analysis correctness

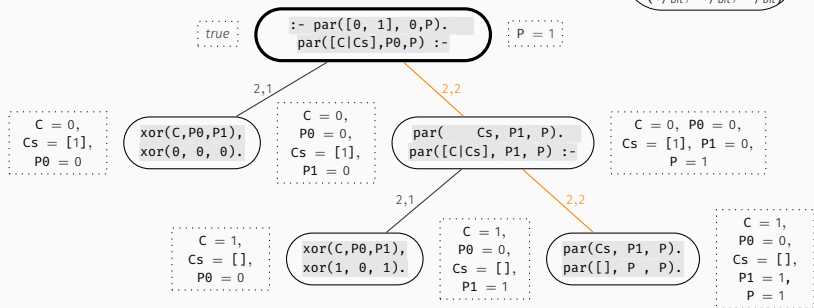
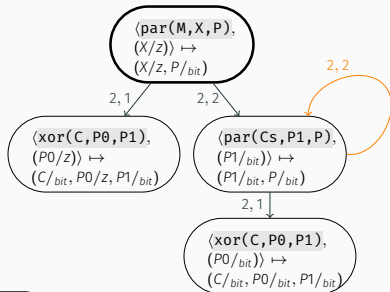
An analysis graph  $\mathcal{A}$  is **correct** for a program  $P$  and a set of queries  $Q$  if it approximates all the calls, **answers**, and dependencies in the concrete semantics  $\llbracket P \rrbracket_Q$  (a set of AND trees).





# Analysis correctness

An analysis graph  $\mathcal{A}$  is **correct** for a program  $P$  and a set of queries  $Q$  if it approximates all the calls, answers, and **dependencies** in the concrete semantics  $\llbracket P \rrbracket_Q$  (a set of AND trees).



# Analysis graphs

Two levels of **abstraction** of program execution:

- control: unbounded number of AND trees as a graph,
- data: parametric abstraction by providing domain operations.

Properties:

- **interprocedural**: each node contains a **summary of the behavior** of a predicate,
- **multivariance**: distinguish different abstract call patterns for
  - precision – differentiate contexts (for optimizations/verification),
  - efficiency – localize recomputation,
- **path approximations**: the edges in a path of the graph abstract the call stack and ordered literals form a **regular approximation of all previously called predicates**,

Note that analysis graphs may be used to analyze **imperative programs**, either via translation to Horn clauses [LOPSTR07] or directly [FTfjP07].

**Input**     $\mathcal{Q}_\alpha$ : initial abstract queries.  
           $P'$ : target program (changed).  
           $\Delta$ : **clauses** that changed from  $P$  to  $P'$ .  
           $\mathcal{A}$ : analysis results of  $P$ .

---

**Output**    a correct analysis graph for  $P'$  and  $\mathcal{Q}_\alpha$ .

The algorithm is based on:

- **events** to trigger **(re)analysis** at the level of **literals**,
- when abstractly executing a predicate call, a **subgraph is reused** if possible,
- for abstract domains requiring **widening**:
  - successive increasing **calls** turned into a **cycle** in the graph,
  - successive increasing **successes** are generalized (**widened**),
  - the **order in event processing** affects the abstract values in analysis result.

## Adding clauses

Expand the graph for the new possible calls (and update answers).

## Deleting clauses

For precision: remove subgraphs and recompute.

# General conditions when restarting an analysis

The following conditions justify all **incremental** algorithms in the thesis, as well as the adding and deleting clauses.

## Starting from a **correct** partial analysis

If an analysis graph correctly abstracts all the behaviors represented in calls of its node, it can be reused to obtain a correct and precise analysis.

## Starting from **any** partial analysis

An analysis graph can be reused to obtain a correct and precise analysis if for any node:

- it correctly abstracts all the behaviors represented by its call, or
- it is scheduled to be reanalyzed.

**Key:** when reusing an analysis result, for efficiency, subgraphs are never (re)checked, only the success of its root node is reused when abstractly executing a literal.

# Incremental and modular analysis



1. Take “**snapshots**” of the program sources (e.g., at each editor save/pause while developing, each commit, ...).
2. **Detect the changes** w.r.t. the previous snapshot.
3. **Reanalyze:**
  - annotate and remove potentially **outdated information**,
  - (re-)analyze **incrementally** module by module until an intermodular fixpoint is reached again.

So far, in abstract interpretation:

- **fine-grain** (clause-level) **incremental analysis** for **non-modular** programs.  
[ICLP'95, SAS'96, Kelly et. al ACSC'97, TOPLAS'00, Albert et. al PEPM'12]
- **coarse-grain** (module-level) analysis aimed at reducing **memory consumption**.  
[Codish et. al POPL'93, ENTCS'00, LOPSTR'01, Cousot & Cousot CC'02]

# Modular logic programs

## Strict module system

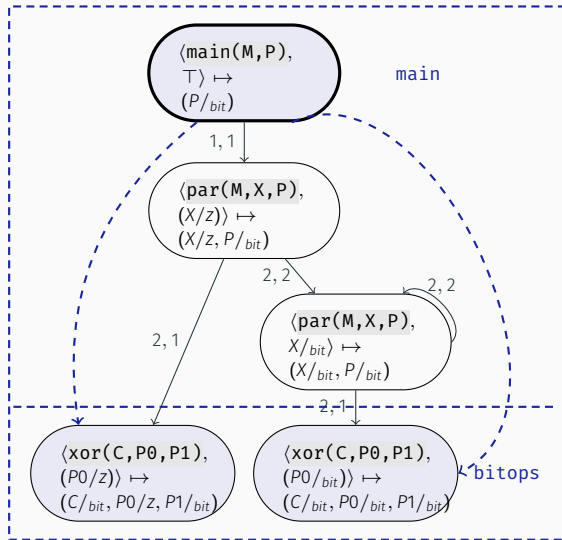
- Modules define an interface of exported and imported predicates.
- Non-exported predicates cannot be seen or used in other modules.

## Modular program

```
1 :- module(main, [main/2]).
2
3 :- use_module(bitops, [xor/3]).
4
5 main(L,P) :-
6     par(L,0,P).
7
8 par([], P, P).
9 par([C|Cs], P0, P) :-
10     xor(C, P0, P1),
11     par(Cs, P1, P).
```

```
1 :- module(bitops, [xor/3]).
2
3 xor(0,0,0).
4 xor(0,1,1).
5 xor(1,0,1).
6 xor(1,1,0).
```

# Analysis graphs for incremental and modular analysis



We have:

- a **global analysis graph**  $\mathcal{G}$ : call dependencies among imported/exported predicates.
- a **local analysis graph**  $\mathcal{L}_M$  per module  $M$ : limited to the predicates used in  $M$ .

# Incremental and modular analysis algorithm

**Input**  $\mathcal{Q}_\alpha$ : initial abstract queries.  
 $\mathbf{P}' (\{M_i\})$ : target program (changed).  
 $\Delta$ : **clauses** that changed from  $\mathbf{P}$  to  $\mathbf{P}'$  (split by module).  
 $\mathcal{A} (\mathcal{G}, \{\mathcal{L}_i\})$ : analysis results of  $\mathbf{P}$ .

---

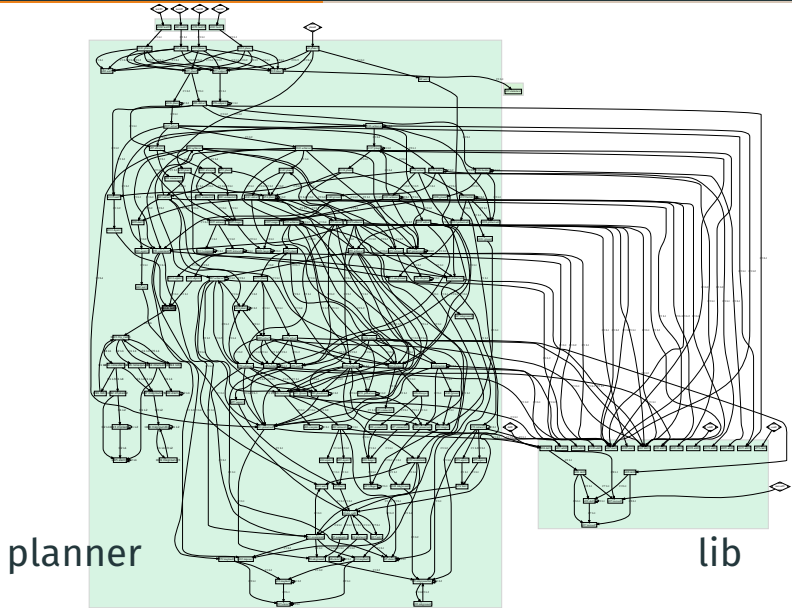
**Output** a correct analysis graph for  $\mathbf{P}'$  and  $\mathcal{Q}_\alpha$ .

The algorithm:

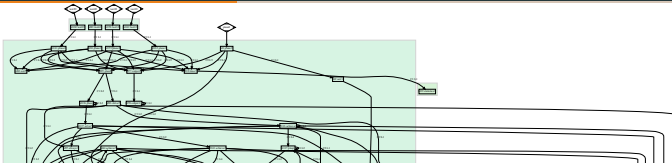
- assumes as answer  $\perp$  when a module has not been analyzed yet,
- (re)starts the analysis of modules using the dependencies in  $\mathcal{G}$ ,
- updates the answers of the imported modules by scheduling new events,
- iterates until an intermodular fixpoint is reached, i.e., the global analysis graph does not change.



# Snapshot of analysis graphs



# Snapshot of analysis graphs



Changes detected!

planner.pl

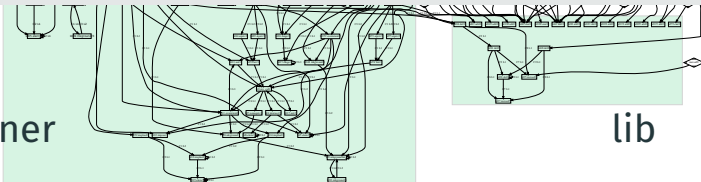
```
100 %%  
101 - explore(P,Map,[P|Map]) :-  
102   - safe(P).  
103 %%
```

lib.pl

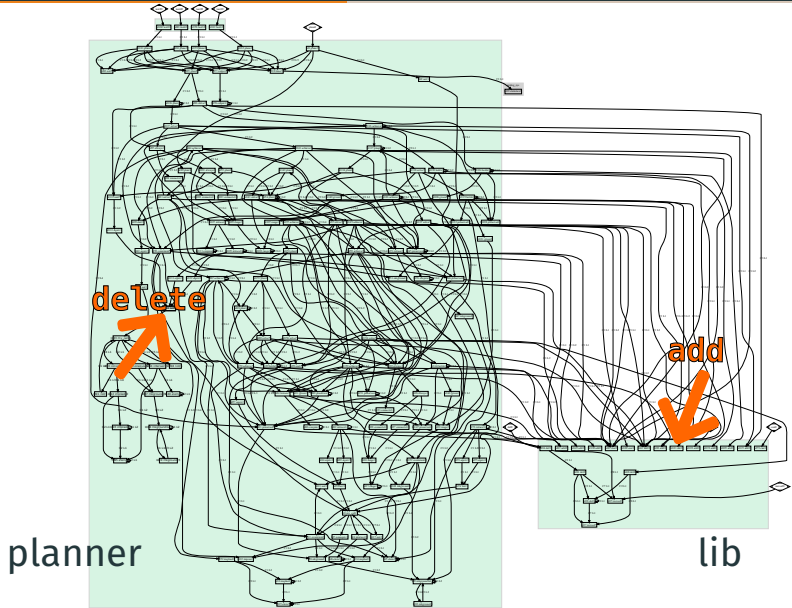
```
41 %%  
42 + add(Node,Graph) :-  
43   +   %% implementation  
44   +   %% implementation  
45 %%
```

planner

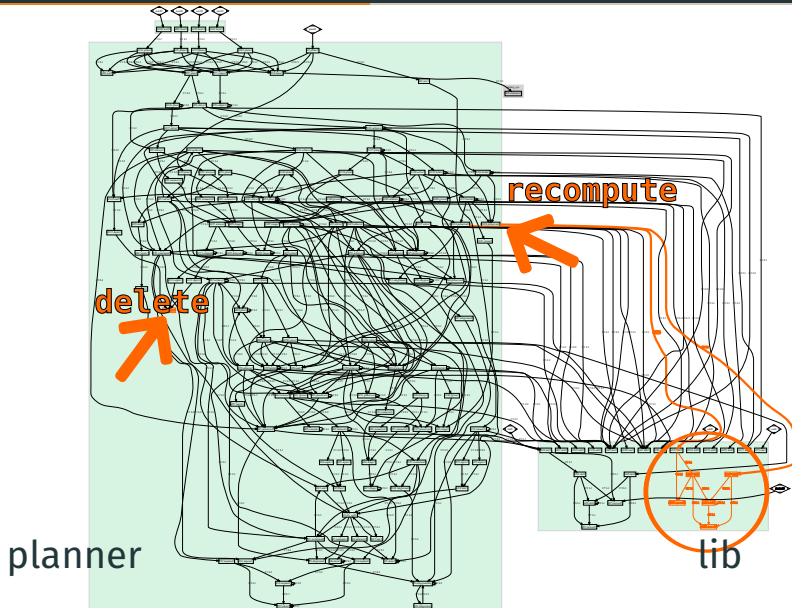
lib



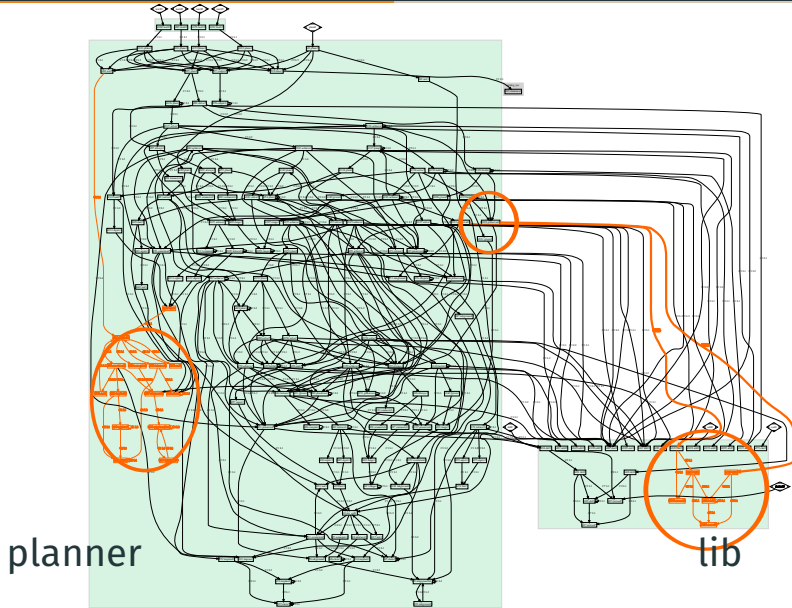
# Snapshot of analysis graphs



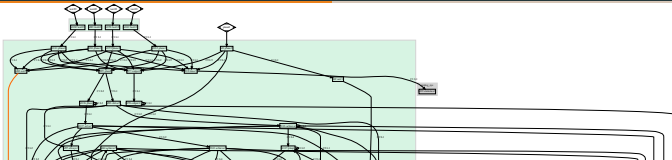
# Snapshot of analysis graphs



# Snapshot of analysis graphs

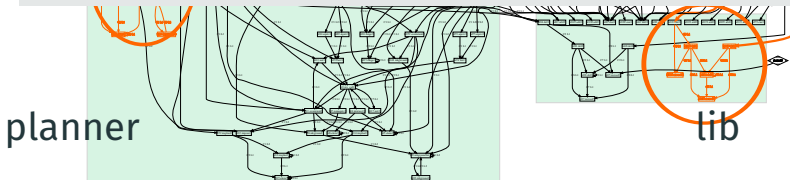


# Snapshot of analysis graphs



## The algorithm:

- maintains local and global graphs for the predicates **and their dependencies**,
- localizes as much as possible fixpoint (re)computation inside modules to minimize context swaps,
- deals incrementally with **additions, deletions**.



# Fundamental results

**Lemma 4.10** (Correctness of INCANALYZE starting from a correct partial analysis). *Let  $P$  be a program,  $Q_\alpha$  be a set of abstract queries, and fix  $q \in Q_\alpha$ . Suppose that  $\mathcal{A}_0$  is the analysis result  $\mathcal{A}_0 = \text{INCANALYZE}(P, Q_\alpha \setminus \{q\}, \emptyset, \emptyset)$ . Then the analysis result  $\mathcal{A} = \text{INCANALYZE}(P, Q_\alpha, \emptyset, \mathcal{A}_0)$  is correct for  $P$  and  $\gamma(Q_\alpha)$ .*

**Theorem 4.11** (Correctness of INCANALYZE starting from a partial analysis). *Let  $P$  be a program,  $Q_\alpha$  a set of abstract queries, and  $\mathcal{A}_0$  a well-formed analysis graph for  $P$ . Suppose for all concrete queries  $q \in \gamma(Q_\alpha)$ , for all nodes  $n$  from which there is a path in the concrete execution  $q \rightsquigarrow n$  in  $[P]Q$ , and for all  $n_\alpha \in \mathcal{A}_0$  such that  $n \in \gamma(n_\alpha)$  either:*

- $n_\alpha \in Q_\alpha$ , or
- the subgraph with root  $n_\alpha$  is correct for  $P$  and  $\{\gamma(n_\alpha)\}$ .

*Then  $\mathcal{A} = \text{INCANALYZE}(P, Q_\alpha, \emptyset, \mathcal{A}_0)$  is correct for  $P$  and  $\gamma(Q_\alpha)$ .*

**Theorem 4.12** (Correctness and precision of INCANALYZE95 starting from a partial analysis). *Under the same conditions as Theorem 4.11, if  $\mathcal{A}_0 \sqsubseteq \mathcal{A}$ , then:*

$$\text{INCANALYZE95}(P, Q_\alpha, \emptyset, \emptyset) = \text{INCANALYZE95}(P, Q_\alpha, \emptyset, \mathcal{A}_0).$$

**Lemma 4.13** (Correctness of INCANALYZE modulo imported predicates). *Let  $M$  be a module of program  $P$ ,  $E$  a set of abstract queries. Let  $\mathcal{L}_0$  be an analysis graph such that  $\forall \langle A, \lambda^c \rangle \in \mathcal{L}_0.\text{mod}(A) \in \text{imports}(M)$ . The analysis result*

$$\mathcal{L} = \text{INCANALYZE}(M, E, \emptyset, \mathcal{L}_0)$$

*is correct for  $M$  and  $\gamma(E)$  assuming  $\mathcal{L}_0$ .*

**Lemma 4.13** (Precision of INCANALYZE modulo imported predicates). *Let  $M$  be a module of program  $P$ ,  $E$  a set of abstract queries. Let  $\mathcal{L}_0$  be an analysis graph such that  $\forall \langle A, \lambda^c \rangle \in$*

*$\mathcal{L}_0.\text{mod}(A) \in \text{imports}(M)$  correctly and precisely approximates the behavior of the imported predicates. The analysis result*

$$\mathcal{L} = \text{INCANALYZE95}(M, E, \emptyset, \mathcal{L}_0)$$

*is the least analysis graph for  $M$  and  $\gamma(E)$  assuming  $\mathcal{L}_0$ .*

**Lemma 5.1** (Correctness updating  $\mathcal{L}$  modulo  $\mathcal{G}$ ). *Let  $M$  be a module of program  $P$  and  $E$  a set of entries. Let  $\mathcal{G}$  be a previous state of the global analysis graph, if  $\mathcal{L}_M$  is correct for  $M$  and  $\gamma(E)$  assuming  $\mathcal{G}$ . If  $\mathcal{G}$  changes to  $\mathcal{G}'$  the analysis result*

$$\mathcal{L}'_M = \text{LOCINCANALYZE}(M, E, \mathcal{G}', \mathcal{L}_M, \emptyset)$$

*is correct for  $M$  and  $\gamma(E)$  assuming  $\mathcal{G}$ .*

**Theorem 5.3** (Correctness of MODINCANALYZE). *Let  $P, P'$  be modular programs that differ by  $\Delta$ ,  $Q_\alpha$  a set of abstract queries, and  $\mathcal{A} = \text{MODINCANALYZE}(P, Q_\alpha, \emptyset, (\emptyset, \emptyset))$ , then if:*

$$\{\mathcal{G}', \{\mathcal{L}'_{M_i}\}\} = \text{MODINCANALYZE}(P', Q_\alpha, \mathcal{A}, \Delta)$$

*$\mathcal{G}'$  is correct for  $P$  and  $\gamma(Q_\alpha)$ .*

**Lemma 5.4** (Correctness and precision updating  $\mathcal{L}$  modulo  $\mathcal{G}$ ). *Let  $M$  be a module contained in program  $P$ ,  $E$  a set of entries. Let  $\mathcal{G}$  be a previous state of the global analysis graph, if  $\mathcal{L}_M = \text{LOCINCANALYZE95}(M, E, \mathcal{G}, \emptyset, \emptyset)$ . If  $\mathcal{G}$  changes to  $\mathcal{G}'$  the analysis result:*

$$\text{LOCINCANALYZE95}(M, E, \mathcal{G}', \mathcal{L}_M, \emptyset) =$$

$$\text{LOCINCANALYZE95}(M, E, \mathcal{G}', \emptyset, \emptyset)$$

*is the same as analyzing from scratch, i.e., the least correct analysis graph of  $M, E$ .*

**Theorem 5.6** (Correctness and precision of MODINCANALYZE). *Let  $P$  and  $P'$  be modular programs that differ by  $\Delta$ ,  $Q_\alpha$  a set of abstract queries, and  $\mathcal{A} = \text{MODINCANALYZE95}(P, Q_\alpha, \emptyset, (\emptyset, \emptyset))$ , then*

$$\text{MODINCANALYZE95}(P', Q_\alpha, \emptyset, (\emptyset, \emptyset)) =$$

$$\text{MODINCANALYZE95}(P', Q_\alpha, \mathcal{A}, \Delta).$$

# Fundamental results

**Lemma 4.10** (Correctness of INCANALYZE starting from a correct partial analysis). Let  $P$  be a program,  $Q_\alpha$  be a set of abstract queries, and fix  $q \in Q_\alpha$ . Suppose that  $\mathcal{A}_0$  is the analysis result  $\mathcal{A}_0 = \text{INCANALYZE}(P, Q_\alpha \setminus \{q\}, \emptyset, \emptyset)$ . Then the analysis result of  $\text{INCANALYZE}(P, Q_\alpha, \emptyset, \mathcal{A}_0)$  is correct for  $P$  and

$\mathcal{L}_0.\text{mod}(A) \in \text{imports}(M)$  correctly and precisely approximates the behavior of the imported predicates. The analysis result

$$\mathcal{L} = \text{INCANALYZE95}(M, E, \emptyset, \mathcal{L}_0)$$

is the least analysis graph for  $M$  and  $\gamma(E)$  assuming  $\mathcal{L}_0$ .

## Contributions

The results from our incremental, modular analysis are:

- **correct over-approximations** of the AND tree semantics, and
- the **least correct analysis graph** if no widening is performed.

Additionally:

- extended traditional algorithm with **widening** (not formalized before),
- **split correctness and precision** of incremental analysis,
- new results **reanalyzing** starting from a **partial analysis**,
- **formalized** results of an **existing modular** algorithm (non incremental).

$$\mathcal{L} = \text{INCANALYZE}(M, E, \emptyset, \mathcal{L}_0)$$

is correct for  $M$  and  $\gamma(E)$  assuming  $\mathcal{L}_0$ .

**Lemma 4.13** (Precision of INCANALYZE modulo imported predicates). Let  $M$  be a module of program  $P$ ,  $E$  a set of abstract queries. Let  $\mathcal{L}_0$  be an analysis graph such that  $\forall (A, \lambda^c) \in$

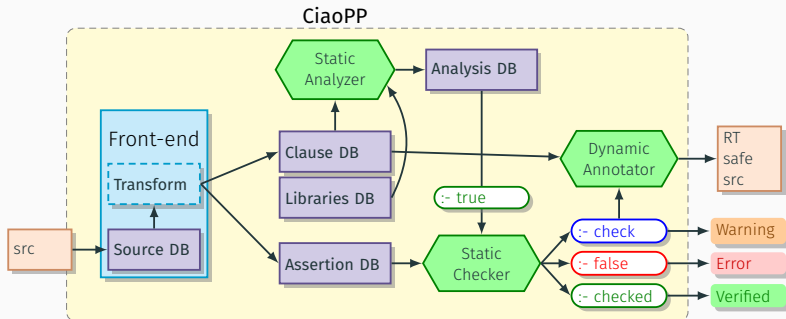
**Theorem 5.6** (Correctness and precision of MODINCANALYZE). Let  $P$  and  $P'$  be modular programs that differ by  $\Delta$ ,  $Q_\alpha$  a set of abstract queries, and  $\mathcal{A} = \text{MODINCANALYZE95}(P, Q_\alpha, \emptyset, (\emptyset, \emptyset))$ , then

$$\text{MODINCANALYZE95}(P', Q_\alpha, \emptyset, (\emptyset, \emptyset)) =$$

$$\text{MODINCANALYZE95}(P', Q_\alpha, \mathcal{A}, \Delta).$$

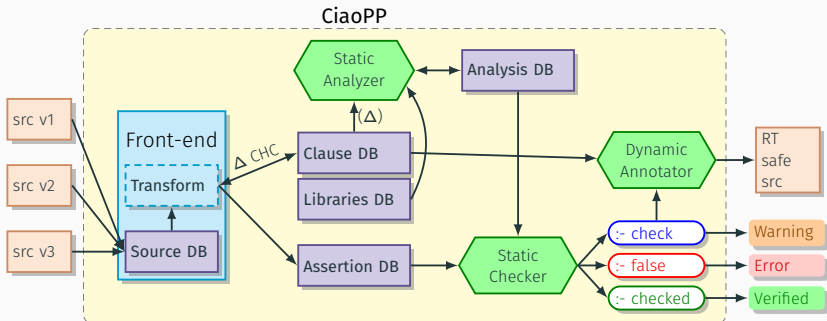


# Implementation: the Ciao model and CiaoPP architecture

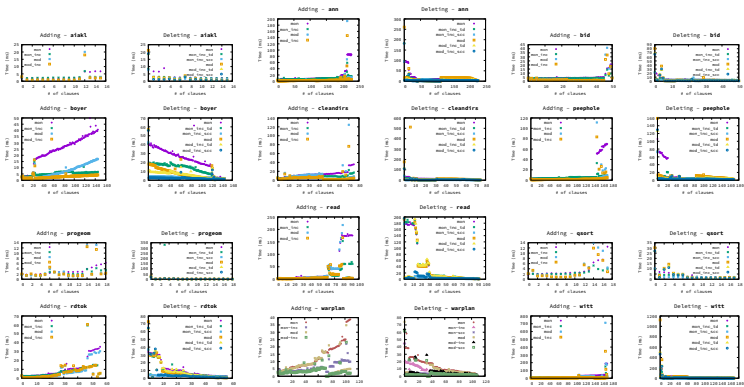
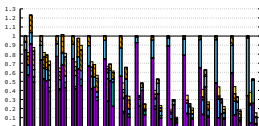
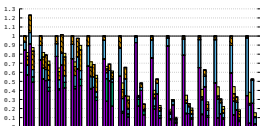
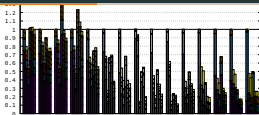
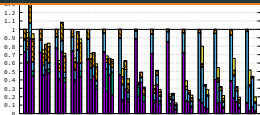


The **Ciao** model is an antecedent to the popular gradual- and hybrid-typing approaches.

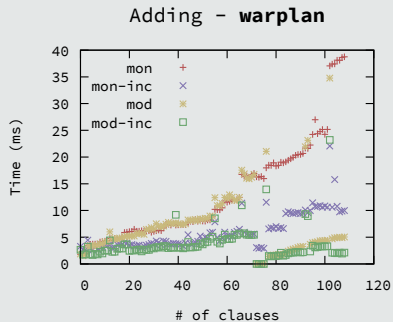
# Implementation: CiaoPP architecture with incrementality



# Experimental evaluation

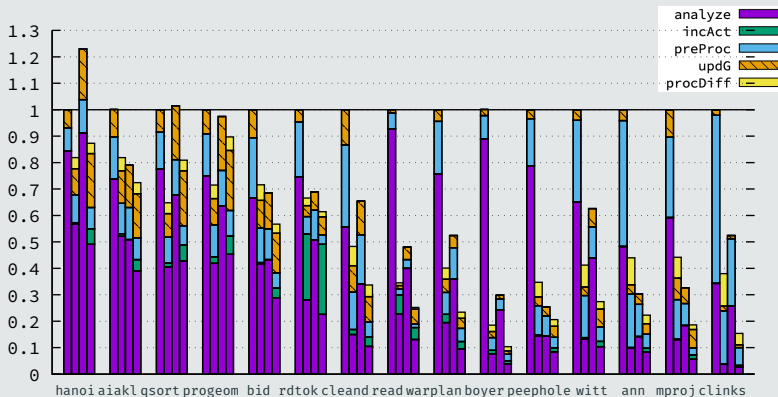


## Addition experiment (time in ms) – def domain



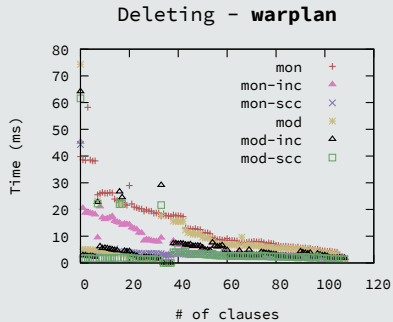
# Experimental evaluation

## Accumulated normalized time (def) – clause addition



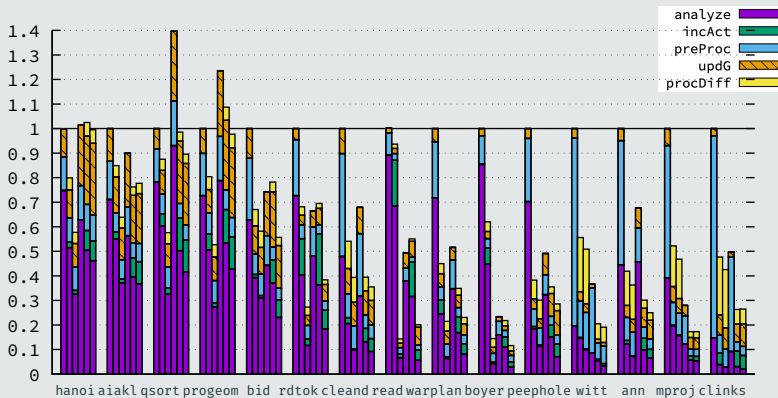
The order inside each set of bars is: |mon|mon\_inc|mod|mod\_inc|.

## Deletion experiment (time in ms) - def domain



# Experimental evaluation

## Accumulated normalized time (def) – clause deletion



The order inside each set of bars is: |mon|mon\_td|mon\_scc|mod|mod\_td|mod\_scc|

## Summary

- almost **immediate** response when the changes do not affect the result,
- up to **13×** overall speedup w.r.t. the original non-incremental algorithm,
- modular analysis from scratch is improved up to **9×**,
- maximum **size** of analysis graphs **reduced**,
- keeping structures for incrementality produces **small overhead**.



# Static on-the-fly verification in CiaoPP

```
42     P = B
43     ; rewrite clause(H,B), clause(H,P), I,G, Info
44     ).
45
46 rewrite( clause(H,B), clause(H,P), I,G, Info) :-
47     numbervars_2(H,0,Lhv),
48     collect_info(B, Info, Lhv, _X, _Y),
49     add_annotations(Info, P, I, G), !.
50
51 :- pred add_annotations(Info, Phrase, Ind, Gnd)
52     : (var(Phrase), indep(Info, Phrase))
53     => (ground(Ind), ground(Gnd)).
54
55 add_annotations([], [], _, _).
56 add_annotations([I|Is], [P|Ps], Indep, Gnd)
57     add_annotations(I, P, Indep, Gnd),
58     add_annotations(Is, Ps, Indep, Gnd).
59
60 add_annotations(Info, Phrase, I, G) :- !,
61     para_phrase( Info, Code, Type, Vars, I, G),
62     make_CGE_phrase( Type, Code, Vars, PCode, I, G),
63     ( var(Code), !,
64       Phrase = PCode
65     ;   Vars = [], !,
66       Phrase = Code
67     ;   Phrase = (PCode, Code)
68     ).
69
```

Verified assertion:

```
:- check calls add_annotations(Info, Phrase, Ind, Gnd)
   : ( var(Phrase), indep(Info, Phrase) ).
```

Verified assertion:

```
:- check success add_annotations(Info, Phrase, Ind, Gnd)
   : ( var(Phrase), indep(Info, Phrase) )
```

← Part of the **parallelizer** code.

# Static on-the-fly verification in CiaoPP

```
42     P = B
43     ; rewrite( clause(H,B), clause(H,P), I,G,Info)
44     ).
45
46 rewrite( clause(H,B), clause(H,P), I,G,Info) :-
47     numbervars_2(H,0,Lhv),
48     collect_info(B,Info,Lhv,_X,_Y),
49     add_annotations(Info,P,I,G),!.
50
51 :- pred add_annotations(Info,Phrase,Ind,Gnd)
52     : (var(Phrase), indep(Info,Phrase))
53     => (ground(Ind), ground(Gnd)).
54
55 add_annotations([],[],_,-).
56 add_annotations([I|Is],[P|Ps],Indep,Gnd)
```

Verified assertion:

```
:- check calls add_annotations(Info,Phrase,Ind,Gnd)
   : ( var(Phrase), indep(Info,Phrase) ).
```

Verified assertion:

```
:- check success add_annotations(Info.Phrase.Ind,Gnd)
```

## Average assertion checking time (s)

Benchmark: chat-80 port – 5.2k LOC across 27 files (Ciao Prolog).

domain	E1			E2			E3		
	noinc	inc	speedup	noinc	inc	speedup	noinc	inc	speedup
pairSh	2.8	1.6	×1.8	2.9	1.5	×1.9	2.8	1.6	×1.8
def	3.0	1.6	×1.9	2.7	1.5	×1.8	2.9	1.7	×1.7
ShGrC	18.1	5.1	×3.5	18.3	5.1	×3.6	18.1	4.5	×4.0

# Static on-the-fly verification in CiaoPP

```
42     P = B
43     ; rewrite(clause(H,B),clause(H,P),I,G,Info)
44     ).
45
46 rewrite( clause(H,B), clause(H,P),I,G,Info) :-
47     numbervars_2(H,0,Lhv),
48     collect_info(B,Info,Lhv,_X,_Y),
49     add_annotations(Info,P,I,G),!.
50
51 :- pred add_annotations(Info,Phrase,Ind,Gnd)
52     : (var(Phrase), indep(Info,Phrase))
53     => (ground(Ind), ground(Gnd)).
54
55 add_annotations([],[],_,-).
56 add_annotations([I|Is],[P|Ps],Indep,Gnd)
```

Verified assertion:

```
:- check calls add_annotations(Info,Phrase,Ind,Gnd)
   : ( var(Phrase), indep(Info,Phrase) ).
```

Verified assertion:

```
:- check success add_annotations(Info.Phrase.Ind,Gnd)
```

## Average assertion checking time (s) – only changing assertions

Benchmark: chat-80 port – 5.2k LOC across 27 files (Ciao Prolog).

domain	E1			E2			E3		
	noinc	inc	speedup	noinc	inc	speedup	noinc	inc	speedup
pairSh	2.8	1.7	×1.6	2.7	1.6	×1.7	2.9	1.7	×1.7
def	3.1	1.5	×2.0	2.9	1.4	×2.0	3.0	1.6	×1.9
ShGrC	18.2	2.0	×9.1	18.1	1.9	×9.6	18.2	1.9	×9.6

# Guiding the analyzer

Two problems that motivate allowing **the user to guide** the analyzer:

1. Automatic **approximations** may lead to **imprecise results**:
  - desired **optimizations** cannot be applied,
  - **assertions** cannot be **verified** (“false alarms”).
2. Analysis may require excessive **resources** (time or space):

Techniques to optionally annotate program parts to **guide invariants inference**:

**Astrée** [Cousot et. al ESOP'05] uses at program point:

- *asserts* with properties that have to be verified,
- *known facts* used to refine abstract state.

**CiaoPP** [ESOP'96] uses assertions that can be qualified with a status:

- *check*: meaning that it needs to be verified,
- *trust*: representing knowledge that the user guarantees to be true (beliefs).

# The Ciao assertion language

**Assertions** express **abstractions of the behavior** of programs.

[ILPSW'97, LP25Y'99]

## pred assertions

```
:- [Status] pred Head [: Pre] [=> Post].
```

- *Head*: predicate that the assertion applies to,
- *Pre*: properties about how the predicate is used (hold when called),
- *Post*: properties that hold if *Pre* holds and the predicate succeeds,
- **Status** qualifies the meaning of assertions.

```
1 :- trust pred fact(N, R)          => (int(N), R > 0).  
2 :- trust pred fact(N, R) : N > 1 => even(R).
```

## Using **trust** assertions

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1  % (y > 0) % Analyzing with an intervals domain (non relational)
2  x = y + 2;
3  % (x > 2, y > 0)
4  z = x - y;
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```
1 % (y > 0)
2 x = y + 2;
3 % (x > 2, y > 0)
4 z = x - y;
5 % (int(z), x > 2, y > 0)
6 trust(z == 2); % Because of line 2
7 % (z = 2, x > 2, y > 0)
```

# Using **trust** assertions

**Trust** assertions may be used to:

- regain precision during analysis.
- speed up computation of analysis.

```
1 :- trust pred html_escape(S0, S) => (string(S0), string(S)).
2 html_escape( "`" | S0, "&ldquo;" | S ) :- !, html_escape(S0, S).
3 html_escape( "'" | S0, "&rdquo;" | S ) :- !, html_escape(S0, S).
4 html_escape( "\"" | S0, "&quot;" | S ) :- !, html_escape(S0, S).
5 html_escape( "'" | S0, "&apos;" | S ) :- !, html_escape(S0, S).
6 html_escape( [X | S0], [X | S] ) :- !,
7     character_code(X),
8     html_escape(S0, S).
9 html_escape( "", "" ).
```

# Using **trust** assertions

**Trust** assertions may be used to:

- **regain precision** during analysis.
- **speed up computation** of analysis.
- define **abstract usage or specifications** of libraries or dynamic predicates.

```
1 :- module(sockets, []).  
2  
3 :- export(receive/2).  
4 :- pred receive(S, M) : (socket(S), var(M)) => list(M, utf8).  
5 :- impl_defined(receive/2).  
6 % receive is written in C
```

# Using **trust** assertions

**Trust** assertions may be used to:

- **regain precision** during analysis.
- **speed up computation** of analysis.
- define **abstract usage or specifications** of libraries or dynamic predicates.
- (re)define the **language semantics** for abstract domains.

```
1 :- trust pred '*'(A, B, C) : (int(A), int(B)) => int(C).  
2 :- trust pred '*'(A, B, C) : (flt(A), int(B)) => flt(C).  
3 :- trust pred '*'(A, B, C) : (int(A), flt(B)) => flt(C).  
4 :- trust pred '*'(A, B, C) : (flt(A), flt(B)) => flt(C).
```

- **Precision:** assertions are precisely applied during analysis:
  - the abstract **success** states inferred are covered by the success assertion conditions (if they exist).
  - the abstract **call** states inferred are covered by the call assertion conditions.
- **Correctness** modulo assertions: the computed analysis is **correct** for  $P, \mathcal{Q}$  if all conditions are **correct**.

# Reacting incrementally to assertion edits

Why? In generic code assertions play a very important role as a place holder for the code that is not available yet, e.g.:

- yet to be implemented,
- compiled in a different language, or
- linked dynamically.

The **contributions** are:

- an **incremental fixpoint algorithm** that reacts to changes in both the program **and the assertions**, and
- an application of this approach to the scalable analysis of generic programming (based on **open predicates**).

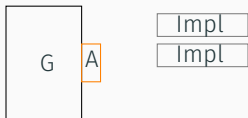
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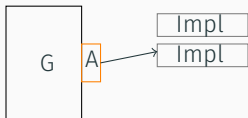
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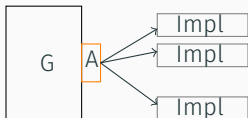
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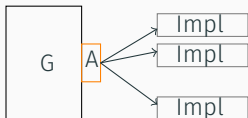
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(And we also proposed an encoding of generic programming in (Ciao) Prolog.)

# Abstract extensionality

How does **the way programs are written** (or transformed) **affect** analysis **precision**?

- **semantically equivalent** programs may exhibit **different properties**.
- **semantically different** programs may **appear identical** when analyzed.
- two classes of programs for a given program  $P$  and a given abstraction  $A$ 
  - **completeness/incompleteness cliques** ( $\mathbb{C}(P, A)/\overline{\mathbb{C}}(P, A)$ ):
    - $\mathbb{C}(P, A)$  is the class of all variants of  $P$  for which the analysis with  $A$  is precise (no false alarms)
    - $\overline{\mathbb{C}}(P, A)$  is the class of all variants of  $P$  for which analysis with  $A$  is imprecise.
    - automatic removal of false alarms is impossible: there is no many-to-one reducibility of  $\overline{\mathbb{C}}(P, A)$  to  $\mathbb{C}(P, A)$
    - we provide systematic reduction of  $\mathbb{C}(P, A)$  into  $\overline{\mathbb{C}}(P, A)$  (obfuscation).

## Incremental and modular analysis

[[ICLP TC'18](#), [TAPAS'19](#), [TPLP'21](#)]

- **theoretical** results of correctness,
- generalized conditions to reuse an analysis graph correctly,
- almost **immediate** response when the changes do not affect the result,
- up to **13×** speedup w.r.t. the original non-incremental algorithm,
- being aware of **modular structures** is useful: up to **2×** speedup when compared with the original incremental algorithm,
- **modular analysis** from scratch is **improved** up to **9×**,
- keeping structures for incrementality produces **small overhead**,
- analyzing **interactively, on-the-fly** is practical! [[F-IDE'21](#), [ICLP'21](#)]

## Guided analysis

[LOPSTR'18]

- we extended the algorithm to **use trusted assertions**,
- we showed how safely assuming assertions affects the analysis result,
- we provided means to **detect incompatible assertions**.

## Incremental analysis with assertions

[LOPSTR'19]

- we extended the context-sensitive analysis algorithm to **react incrementally** to fine grain changes in (multivariant) **assertions**,
- we showed an application for generic code, to efficiently **specialize the analysis result** as implementations become **available**,
- we provided a syntax to build generic programs in Prolog using **traits**.

## Abstract extensionality

[POPL'20]

- non-trivial abstract interpretation always unveils implicitly also properties concerning the way programs are written,
- automatic **semantic code obfuscation**,
- systematic **removal of false alarms** is **impossible**,
- the class of all programs that are incomplete for a given non-trivial abstraction is **Turing complete**.

- Applications of incremental and modular analysis:
  - improving performance when **combining analysis** with other techniques (e.g., transformations),
  - **semantic code search**,
  - **parallel fixpoint computation**,
- **Heuristics** for automatic configuration of incrementality settings.
- **Amenability** of abstract domains **to incrementality**.
- **Incrementality-aware transformations** (from other source languages).

# A scalable static analysis framework for reliable program development exploiting incrementality and modularity

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PhD Thesis Defense  
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